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FOR SPACE INTERCEPTION

by

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DESIGN OF NEAR-OPTIMAL GUIDANCE LAW FOR SPACE INTERCEPTION

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ABSTRACT: The article applies an augmented-linearization method for nonlinear systems to propose a near-optimal control resolution method of secondary type index problems for nonlinear systems. This method is applied to design a near-optimal terminal guidance law for space interception. As indicated in the simulation results, the near-optimal guidance has good performance.

Key Words: 1. terminal guidance, 2. optimal guidance law.

I. Introduction

Classical optimal control theory has been relatively perfect; however, many difficulties will be confronted when using these theories to solve some concrete optimal control problems. In particular, when a system is nonlinear, almost no analytical form of optimal control can be attained. Therefore, it is necessary to seek the applicable optimal control solution method for nonlinear systems.

Based on the foregoing reasons, the article studies the optimal control problems of a type simulation nonlinear system in the secondary index significance. By applying the augmented-linearization method for nonlinear systems, a near-optimal control solution method is proposed. As an application example, this near-optimal control solution was applied to design a terminal-guidance law in space interception. With simulation comparison against the optimal guidance law, the results show good effects.

II. Suggestions of the Problem

Assume a nonlinear control system

$$\dot{X}(t)=f(X(t))+BU(t), X(t_0)=X. \tag{2—1}$$

In the equation, $X \in R^{n \times 1}, U \in R^{m \times 1}, T \in [t_0, t_f] \subset R, B$, B is an nxm-dimensional constant matrix; f is a continuous function of x. There exists an N+1 order continuous partial derivative with respect to Xi (i=1,2,...,n) for f. N is a fixed natural number, and it is assumed that f(0)=0. For these systems of f(0) not equal to 0, it is very easy to transform to f(0)=0.

Now it is required to find the optimal control vector U(t) so that the index

$$J=\frac{1}{2}X^T(t_f)SX(t_f)+\frac{1}{2}\int_{t_0}^{t_f}[X^T(t)QX(t)+U^T(t)RU(t)]dt \tag{2—2}$$

is the smallest. S, Q, and R are, respectively, the steady weighted matrixes of nxn dimension, nxn dimension, and mxm dimension. Moreover, S and Q are nonnegative steady matrices, and R is a positive steady matrix.

III. Augmented Linearization of Nonlinear Systems

By introducing the row vectors of 1 to N order monomials not related to all linearization, as derived from vector

$X=[X_1X_2...X_m]^T$ from elements Xi (i=1, 2, ..., n),

$$\Omega \triangleq [X_1, X_2, \cdots, X_1^2, X_1, X_2, \cdots, X_1^N, \cdots, X_n^N]^T \in R^{L\times 1} \quad (3-1)$$

In the equation, $L=\sum_{i=1}^{N} C_{n+i-1}^{i}$, and assume U=-KΩ (3-2)

In the equation, $K \in R^{m\times L}$, then Eq. (2-1) becomes

$$\dot{X}=f(X)-BK\Omega \triangleq g(K,X) \quad (3-3)$$

By deriving both sides of Eq. (3-1) with respect to t, we obtain

$$\begin{aligned}\dot{\Omega} &= [\dot{X}_1, \dot{X}_2 \cdots, NX_n^{N-1}\dot{X}_n]^T \\ &[g_1(K,X), g_2(K,X), \cdots, \\ &NX_n^{N-1}g_n(K,X)]^T \\ &\triangleq G(K,X) \quad (3-4)\end{aligned}$$

Since the Taylor formula applies $G_k(K,X)$ (k=1, 2, ..., L), there is

$$G_k(K,X)=G_k(K,O)+\sum_{i=1}^{N}\frac{1}{j!}[X_1\frac{\partial}{\partial X_1}+X_2\frac{\partial}{\partial X_2}+\cdots+X_n\frac{\partial}{\partial X_n}]^j G_k(K,O) + \frac{1}{(N+1)_1}[X_1\frac{\partial}{\partial X_1}+X_2\frac{\partial}{\partial X_2}+\cdots+X_n\frac{\partial}{\partial X_n}]^{N+1}G_k(K,\theta X) \quad (3-5)$$

In the equation, $\theta \in (0.1) \subset R$, therefore Eq. (3-4) can be written as

$$\dot{\Omega}(t)=b(K)+A(K)\Omega(t)+\varnothing_{N+1}(t) \quad (3-6)$$

in the equation,

$$\varnothing_{N+1}(t)=\sum_{r_1+\cdots+r_n=N+1}\frac{1}{r_1!\cdots r_n!}\cdot\frac{\partial^{r_1+\cdots+r_n}G(K,X)}{\partial X_1^{r_1}\cdots\partial X_n^{r_n}}\Bigg|_{X^r=\prod_{i=1}^{n}X_i^{r_i}(t)}$$

In the equation, $\zeta=\theta X$. Because when X=0, Ω=0, and f(0)=0, therefore, g(K,0)=f(0)-BK·0=0. Then from Eq. (3-4) we can see Gk(K,0)=0, (k=1,2,..., L). Therefore, from Eq. (3-5) we know that b(K)=0 in Eq. (3-6). Therefore, we have

$$\dot{\Omega}(t)=A(K)\Omega(t)+\varnothing_{N+1}(t) \qquad (3\text{-}7)$$

If we neglect the term $\varnothing_{N+1}(t)$, then

$$\dot{Z}(t)=A(K)Z(t),Z(t_o)=\Omega(t_o) \qquad (3-8)$$

In the equation, $Z\in R^{L\times 1}$. The foregoing equation is the augmented linear system of system (2-1) obtained with N-order Taylor expansion.

IV. Augmented Linearized Error Analysis

There is a relationship between the status vectors of nonlinear system (2-1) and augmented linear system (3-8)

$$X(t)=[I_{n\times n}|O_{n\times(L-n)}]Z(t) \qquad (4\text{-}1)$$

Subtract Eq. (3-8) from Eq. (3-7), and integrate, then we obtain

$$\Omega(t)-Z(t)=\int_{t_o}^{t}H(t,\tau)\varnothing_{N+1}(\tau)d\tau \quad (4-2)$$

In the equation, the status transfer matrix H(t,τ) satisfies

$$\begin{cases}\dfrac{\partial H(t,\tau)}{\partial t}=A(K)H(t,\tau) & (4\text{-}3a)\\ H(t,t)=I & (4\text{-}3b)\end{cases}$$

Therefore, we can obtain the status approximate error

$$|X_i(t)-Z_i(t)|\leqslant D(t)\,\|X\|^{N+1} \qquad (4-4)$$

In the equation, D(t) is

$$D(t)=\int_{t_0}^{t}\left|\ \varphi_1^T H\ (t,\ \tau)\ \sum_{r_1+\cdots+r_n=N+1}\right.$$
$$\frac{1}{r_1!\cdots r_n!}\cdot\frac{\partial^{r_1+\cdots+r_n}G(k,X)}{\partial X_1^{r_1}\cdots\partial X_n^{r_n}}|_{X=\zeta(\tau)}\,|d\tau$$

However, in the equation $\|X\|$ is defined as

$$\|X\|=\mathrm{Sup}\left\{\begin{matrix}|Xi(\tau)|:\\ t_0\leqslant\tau\leqslant t,\ 1\leqslant i\leqslant n\end{matrix}\right\}$$
$$\varphi_1^T=[0\cdots0\,\vdots\,0\cdots0]$$

Assume that D(t) has a boundary in the convex domains including X and O, then we have

$$|X_i(t)-Z_i(t)|=o(\|X\|^{N+1})\ (i=1,\ 2,\cdots,n)\qquad(4-5)$$

V. Near-optimal Control of Nonlinear Systems

Eq. (3-8) can always be written as:

$$\dot{Z}(t)=\tilde{A}(K)Z(t)-\tilde{B}KZ(t)\qquad(5\text{-}1)$$

In the equation, $\tilde{B}=\begin{bmatrix}B\\ O\end{bmatrix}$. Since Z is an approximation of Ω, Eq. (3-2) can be approximated as

$$U=-KZ\qquad(5-2)$$

Substitute the foregoing equation in Eq. (5-1) and we obtain

$$\dot{Z}(t)=\tilde{A}(K)Z(t)+\tilde{B}U(t)\qquad(5-3)$$

In the equation, $Z\in R^{L\times 1}$, and $\tilde{Z}(to)=[X_1(to),\cdots,X_n^N$,
now the performance index (2-2) can be made equivalent to

$$J=\frac{1}{2}Z^T(t_f)\tilde{S}Z(t_f)+\frac{1}{2}\int_{t_0}^{t_f}[Z^T(t)\tilde{Q}Z(t)$$
$$+U^T(t)RU(t)]dt\qquad(5-4)$$

In the equation

$$\tilde{S}=\begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix}, \tilde{Q}=\begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}$$

Eqs. (5-3) and (5-4) describe a linear secondary type optimal adjuster problem. The solution of the problem is Eq. (5-2). In the equation, K is

$$K=R^{-1}\tilde{B}^{T}P \qquad (5-5)$$

and P satisfies the Riccati equation

$$\dot{P}=-P\tilde{A}(K)-\tilde{A}^{T}(K)P+P\tilde{B}R^{-1}P-\tilde{Q} \qquad (5-6)$$

Therefore, the near-optimal control of the original nonlinear equation is

$$U=-R^{-1}\tilde{B}^{T}PZ \qquad (5-7)$$

VI. In designing a terminal-guidance law for space interception, the space interception dynamics equation in the longitudinal-direction plane is [1]

$$\begin{cases} \dot{r}=V & (6\text{-}1a) \\ \dot{V}=r\omega^{2} & (6\text{-}1b) \\ \dot{\omega}=-\dfrac{2V\omega}{r}-\dfrac{a}{r} & (6\text{-}1c) \end{cases}$$

In the equations, r indicates the relative distance between interceptor and target; V stands for the relative velocity; ω indicates the angular rate of the line-of-sight; and a indicates the dynamic acceleration of the interceptor. From the foregoing equation, with some manipulations, we obtain

$$\frac{d(rV\omega^{2})}{dt}=\frac{1}{V^{2}}(rV\omega^{2})^{2}-(3V^{2}\omega^{2}+2V\omega a) \qquad (6\text{-}2)$$

From this equation, by introducing the status variable and the control variable,

$$X(t)=rV\omega^{2} \qquad (6-3a)$$

$$u(t)=3V^{2}\omega^{2}+2V\omega a \qquad (6-3b)$$

Then, Eq. (6-2) becomes a first-order nonlinear system.

$$\dot{X}=\frac{1}{V^2}X^2-u \qquad (6-4)$$

In the equation, V can be considered as a constant because in the terminal guidance phase of space interception, generally the orbital parameters of target motion are known. The interceptor aligns with the target in flying along the predetermined trajectory, and the relative velocity between interceptor and target varies little during the entire terminal-guidance phase; fundamentally, this is a known constant.

The basis for designing the terminal-guidance law is as follows: let the angular rate ω

of the line of sight approach zero, and gradually approach the guidance in parallel. In this article, the guidance law is designed in this manner: first, an index is presented on the system (6-4)

$$J=\frac{1}{2}SX^2(t_f)+\frac{1}{2}\int_{t_0}^{t_f}[qX^2(t)+\mu u^2(t)]dt \qquad (6-5)$$

Then, the optimal control u(t) is derived so that J--->min. Then from Eq. (6-3b), the guidance law is derived. Thus, we have $\omega \to 0$.

In the following, we solve for the problems (6-4) and (6-5) for near-optimal control. Here, we take N=2. In other words, the system (6-4) is augmented linearized as

$$\dot{Z}=\begin{bmatrix} k_1 & k_2+\frac{1}{V^2} \\ 0 & 2k_1 \end{bmatrix} Z \qquad (6-6)$$

In the equation, $Z=[x,x^2]^T$. Since

$$Z=[x,x^2]^T, \text{由于}$$
$$u(t)=-KZ=-k_1x-k_2X^2 \qquad (6-7)$$

Therefore, Eq. (6-6) can be written as

$$\dot{Z}=\tilde{A}(K)Z+\tilde{B}u \qquad (6-8)$$

In the equation, the coefficient matrix

$$\tilde{A}(K)=\begin{bmatrix} O & \frac{1}{V^2} \\ O & 2k_2 \end{bmatrix}, \tilde{B}=\begin{bmatrix} -1 \\ O \end{bmatrix}, K=$$

$[k_1k_2]$

By solving for the Riccati equation [2],

$$\begin{cases} -\dot{P}(t)=P(t)\tilde{A}(K)+\tilde{A}^T(K)P(t)- \\ P(t)\tilde{B}\frac{1}{\mu}\tilde{B}^TP(t)+\tilde{Q} \\ P(t_f)=\tilde{S} \end{cases} \qquad (6-9)$$

In the equation,

$$\tilde{Q}=\begin{bmatrix} q & O \\ O & O \end{bmatrix}, \tilde{S}=\begin{bmatrix} S & O \\ O & O \end{bmatrix}$$

the derived P(t) is substituted into

$$K=[-\frac{1}{\mu} \quad O]P(t)$$

Finally, we obtain

$$k_1=-\sqrt{\frac{q}{\mu}} \frac{\sqrt{q\mu}Sh(\sqrt{\frac{q}{\mu}}t)+S\cdot Ch(\sqrt{\frac{q}{\mu}}t)}{\sqrt{q\mu}ch(\sqrt{\frac{q}{\mu}}t)+S\cdot Sh(\sqrt{\frac{q}{\mu}}t)} \qquad (6-10a)$$

$$k_2=\frac{2k_1^2\mu-q}{V^2(q-4k_1^2\mu)}+$$

$$\frac{\sqrt{q}-k_1\sqrt{\mu}}{2V^2(\sqrt{q}-2k_1\sqrt{\mu})}\exp(\sqrt{\frac{q}{\mu}}-2k_1)t$$

$$+\frac{\sqrt{q}+k_1\sqrt{\mu}}{2V^2(\sqrt{q}+2k_1\sqrt{\mu}}\exp-(\sqrt{\frac{q}{\mu}}+2k_1)t$$

$$(6-10b)$$

Substitute the two foregoing equations in Eq. (6-7) and obtain the near-optimal control of the problems (6-4) and (6-5). Further, based on Eq. (6-3b) we can derive the near-optimal guidance law

$$a(t)=-\frac{3}{2}V\omega(t)-\frac{k_1(t)}{2}r(t)\omega(t)-$$

$$\frac{k_2(t)}{2}r^2(t)V\omega^3(t) \qquad (6-11)$$

VII. Numerical Simulation

By using the following parameters as an example for numerical simulation: S=0.4, q=0.4, μ=0.2, r(0)=150km, V=-7.5km/s, $\omega(0)=5\times10^{-4}$rad/s. Then the initial condition of system (6-4) is X(0)=-281.25(m^2/s^3 [sic]). Figs. 1 and 2 indicate the solution curves of optimal control problems (6-4) and (6-5). Solid lines represent the near-optimal control derived by using the method in the article; the dashed lines indicate the optimal control derived from the numerical method by using the NPSOL software package. Curves in the figures indicate that the near-optimal control is relatively close to the optimal control.

Figs. 3 to 6 indicate the comparative relationship between the near-optimal guidance law (represented by solid lines) derived by using the method in this article, and the corresponding optimal guidance law (represented by the dashed lines) derived by using the NPSOL software package.

From these figures, we can see that they are relatively close to each between the optimal guidance law and the near-optimal guidance law. Additionally, two physical quantities of relative distance and relative velocity of interceptor and target exhibit basically no difference under optimal control and near-optimal control.

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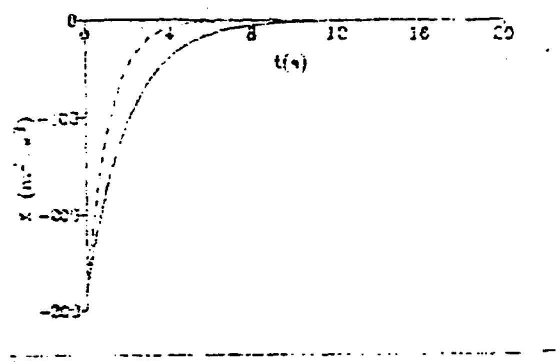

Fig. 1. Optimal and near-optimal status

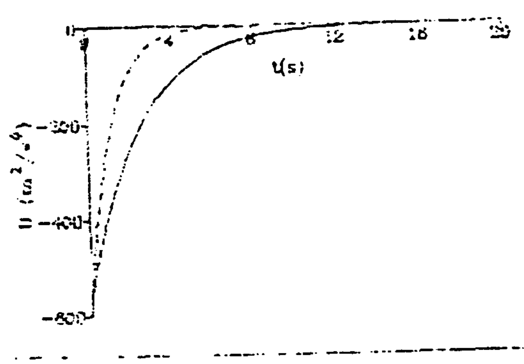

Fig. 2. Optimal and near-optimal control

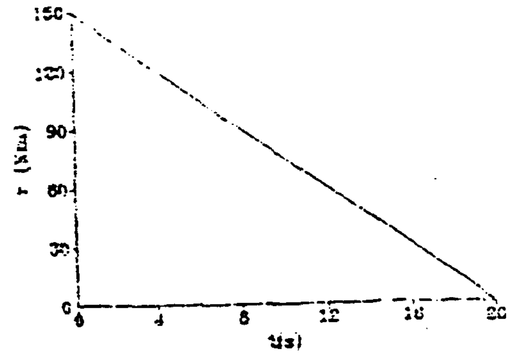

Fig. 3. Relative distance between interceptor and target

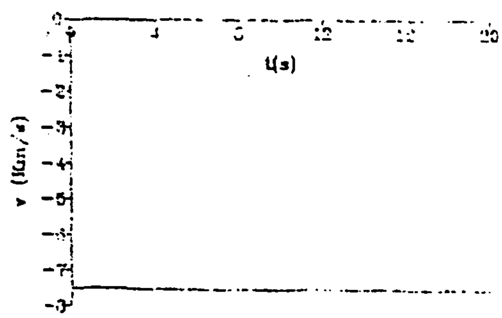

Fig. 4. Relative velocity between interceptor and target

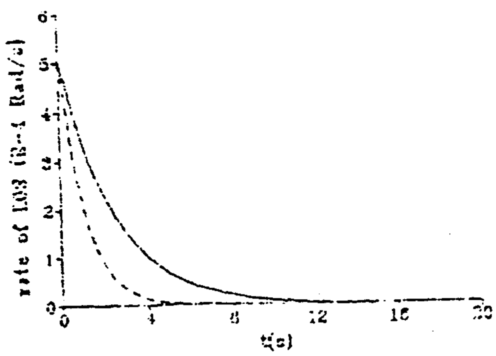

Fig. 5. Variation of angular rate of line of sight Rate of LOS indicates the angular rate of line of sight

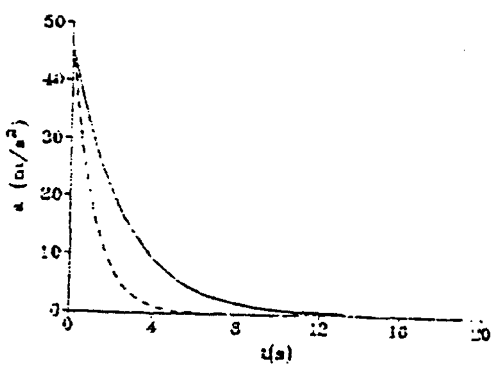

Fig. 6. Variation of powered acceleration

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